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THE DYNAMIC STRESS OF A CONDUCTING HALF-SPACE WITH A CURVILINEAR CUT IN A STRONG MAGNETIC FIELD (ANTIPLANAR DEFORMATION STATE)†

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During the magnetic excitation of a dia(para)magnetic material of a body which is located in a static magnetic field, induced (vortex) currents are generated in the body, leading to the appearance of bulk Lorentz forces. When allowance is made for these forces, an additional tensor is obtained, the Maxwellian stress tensor, which introduces considerable corrections to the stressed state of the body. A magneto-elastic boundary-value problem is considered below for a half-space which is attenuated by a curvilinear cavity-cut. The problem reduces to a singular integral equation. Results of calculations are presented which characterize the dependence of the stress intensity factor, $K_{\rm III}$, on the configuration of the cut, the strength of the applied magnetic field and the excitation frequency. An analogous problem for a single rectilinear cut in an unbounded dia(para)magnetic medium has been studied in [1].

1. BASIC RELATIONSHIPS OF LINEAR MAGNETO-ELASTICITY. FORMULATION OF THE PROBLEM

THE COMPLETE system of equations of magneto-elasticity has the form [1–3]

$$\operatorname{rot}\mathbf{E}+\mathbf{B}^{\cdot}=0,\ \operatorname{rot}\mathbf{H}-\mathbf{D}^{\cdot}=j,\ \operatorname{div}\mathbf{D}=\rho_e,\ \operatorname{div}\mathbf{B}=0 \tag{1.1}$$

$$\partial_j\sigma_{ij}+\rho_eE_i+(\mathbf{j}\times\mathbf{B})_i=\rho u_i^{\cdot\cdot} \tag{1.2}$$

$$\begin{gathered}\mathbf{D}=\varepsilon\mathbf{E}+\alpha(\mathbf{v}\times\mathbf{H}),\ \mathbf{B}=\mu_e\mathbf{H}-\alpha(\mathbf{v}\times\mathbf{E})\\ j_i=\rho_e\mathbf{v}+\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B}),\ \alpha=\varepsilon\mu_e-\varepsilon_0\mu_0,\ \mathbf{v}=\mathbf{u}^{\cdot}\\ \sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\delta_{ij}\varepsilon_{kk},\ \varepsilon_{ij}={}^1/_2(\partial_ju_i+\partial_iu_j),\ \partial_i=\partial/\partial x_i\end{gathered} \tag{1.3}$$

$$\begin{gathered}[\mathbf{E}+\mathbf{v}\times\mathbf{B}]_\tau=0,\ [\mathbf{H}-\mathbf{v}\times\mathbf{D}]_\tau=0\\ [\mathbf{B}]_n=0,\ [\mathbf{D}]_n=0,\ [\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B})+\rho_e\mathbf{v}]_n=0\\ [\sigma_{ij}+t_{ij}]\,n_j=0\ (i,j,k=1,2,3)\\ t_{ij}=E_iD_j+H_iB_j-{}^1/_2\delta_{ij}(E_kD_k+H_kB_k)\end{gathered} \tag{1.4}$$

Relations (1.1) are Maxwell's equations, (1.2) are the equations of motion, (1.3) are the material equations, (1.4) are the boundary conditions on the surface of separation of the media and **E**, **H** and **D**, **B** are the strengths and fluxes of the electrical and magnetic fields, respectively, ε, ε_0 and μ, μ_0 are the permittivities and magnetic permeabilities of the substance and of a vacuum, ρ_e is the electric space charge density, **j** is the current density, ρ is the density of the substance, u_i and σ_{ij} are the mechanical strains and stresses, t_{ij} are the Maxwellian stresses, μ and λ are Lamé constants and δ_{ij} is the Kronecker delta. The square brackets denote jumps in the corresponding quantity on the line of separation of the media.

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Let a strong magnetic field $\mathbf{H}°$ act in a magnetic medium which is in a state of rest. An external perturbation gives rise to a deformation of the body and, correspondingly, to an electromagnetic field which can be described by the small fluctuations $\mathbf{e}$ and $\mathbf{h}$. We shall henceforth assume that this is quasistatic [3].

In the case of such materials as aluminium and copper, it is advisable to simplify the model by ascribing an ideal conductivity ($\sigma \to \infty$) to the medium. In this case, by putting $\mathbf{H} = \mathbf{H}° + \mathbf{h}$ and $\mathbf{E} = \mathbf{e}$, instead of (1.1)–(1.4), we obtain the following system of equations and boundary conditions:

$$
\begin{gathered}
\mathbf{h} = \operatorname{rot}\,(\mathbf{u} \times \mathbf{H}°),\ \mathbf{e} = -\mu_e\,(\mathbf{u}^{\cdot} + \mathbf{H}°) \\
\mu\nabla^2\mathbf{u} + (\lambda + \mu)\,\operatorname{grad}\operatorname{div}\mathbf{u} + \mu_e\,(\operatorname{rot}\mathbf{h}) \times \mathbf{H}° = \rho\mathbf{u}^{\cdot\cdot} \\
[\mathbf{h}]_\tau = 0,\ [\mu_e\mathbf{h}]_n = 0 \\
[\sigma_{ij} + \mu_e\,(H_i{}^\circ h_j + H_j{}^\circ h_i - \delta_{ij}H_k{}^\circ h_k)]\, n_j = 0
\end{gathered}
\tag{1.5}
$$

Let us now assume that the magneto-elastic medium is inhomogeneous: in it there are tunnel cuts along the x_3 axis, L_j ($j = 1, 2, \ldots, k$) and the vector of the initial magnetic field $\mathbf{H}° = (0, H_0, 0)$.

The corresponding static field (which is independent of the x_3 coordinate) is described by the following system of equations and boundary conditions:

$$
\begin{gathered}
\mu\nabla^2 u_i{}^\circ + (\lambda + \mu)\,\partial_i\theta_0 = 0 \quad (i = 1,\ 2) \\
\mu\nabla^2 u_3{}^\circ = 0,\ \nabla^2 = \partial_1{}^2 + \partial_2{}^2 \\
\theta_0 = \partial_1 u_1{}^\circ + \partial_2 u_2{}^\circ \\
\sigma_{1i}{}^\circ n_1 + \sigma_{i2}{}^\circ n_2 = X_{in} + {}^1/_2\varkappa\mu_0 H_0{}^2\,(1 + \varkappa\sin^2\psi)\, n_i \\
\sigma_{31}{}^\circ n_1 + \sigma_{32}{}^\circ n_2 = X_{3n} \\
H_1{}^* = {}^1/_2\varkappa H_0\sin 2\psi,\ H_2{}^* = H_0\,(1 + \varkappa\sin^2\psi) \\
n_1 = \cos\psi,\ n_2 = \sin\psi,\ \varkappa = \mu_e/\mu_0 - 1
\end{gathered}
\tag{1.6}
$$

Here X_{in} ($i = 1, 2$) are the corresponding components of the mechanical stress vector on the edges of L_j, μ_e and μ_0 are the magnetic permeabilities of the material and of the substance which occupies the cavity of the crack (vacuum), ψ is the angle between the normal to the left edge of L_j (when moving from the beginning a_j to the end b_j) and the x_1 axis (Fig. 1), the superscript degree denotes the components of the static field and an asterisk refers to the cavity of the crack.

Hence, the static field is subdivided into a state of plane deformation and a state of antiplane deformation. It should also be noted that the magnetic permeabilities of many dia(para)magnetic substances are practically identical to the magnetic permeability of a vacuum μ_0. It is therefore possible to put $x = 0$, which substantially simplifies the formation of the boundary conditions.

Furthermore, by virtue of the relationships

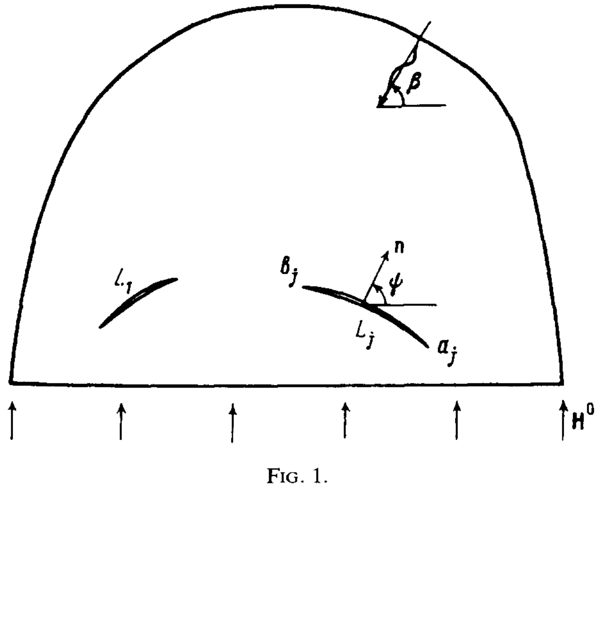

FIG. 1.

$$\mathbf{h} = \mathbf{rot}\,(\mathbf{u} \times \mathbf{H}^\circ) = (H_0\partial_2 u_1,\ -H_0\partial_1 u_1,\ H_0\partial_2 u_3)$$
$$(\mathbf{rot}\,\mathbf{h}) \times \mathbf{H}^\circ = (H_0^2\nabla^2 u_1,\ 0,\ H_0^2\partial_2^2 u_3)$$

we obtain the complete system of equations and the boundary conditions which determined the field of fluctuations.

Plane deformation:

$$(1+\chi^2)\nabla^2 u_1 + \sigma_*\partial_1\theta = c_2^{-2}\ddot{u}_1 \tag{1.7}$$
$$\nabla^2 u_2 + \sigma_*\partial_2\theta = c_2^{-2}\ddot{u}_2,\quad c_2 = \sqrt{\mu/\rho}$$
$$\theta = \partial_1 u_1 + \partial_2 u_2,\quad \chi^2 = \mu_e H_2^2/\mu,\quad \sigma_* = (\mu+\lambda)/\mu$$

$$h_1 = H_0\partial_2 u_1,\quad h_2 = -H_0\partial_1 u_1,\quad h_3 = 0 \tag{1.8}$$
$$e_1 = e_2 = 0,\quad e_3 = -\mu_e H_0 \dot{u}_1$$

$$\sigma_{11}\cos\psi + \sigma_{12}\sin\psi = \mu_0\varkappa H_0\{h_2 + (h_1\cos\psi + h_2\sin\psi)\varkappa\sin\psi\}\cos\psi + X_{1n}$$
$$\sigma_{21}\cos\psi + \sigma_{22}\sin\psi = \mu_0\varkappa H_0(h_1\cos\psi + h_2\sin\psi)(1+\varkappa\sin^2\psi) + X_{2n} \tag{1.9}$$
$$h_1^* = h_1(1+\varkappa\cos^2\psi) + \varkappa h_2\sin\psi\cos\psi$$
$$h_2^* = h_2(1+\varkappa\sin^2\psi) + \varkappa h_1\sin\psi\cos\psi$$

[(1.7) are the equations of motion, (1.8) are the components of the electromagnetic field, and (1.9) are the boundary conditions on L_j ($j = 1, 2, \ldots, k$)].

Antiplane deformation:

$$\nabla^2 u_3 + \chi^2\partial_2^2 u_3 = c_2^{-2}\ddot{u}_3 \tag{1.10}$$
$$h_1 = h_2 = 0,\quad h_3 = H_0\partial_2 u_3 \tag{1.11}$$
$$e_1 = -\mu_e H_0\dot{u}_3,\quad e_2 = e_3 = 0$$
$$\sigma_{31}\cos\psi + \sigma_{32}\sin\psi = X_{3n},\quad h_3^* = h_3 = H_0\partial_2 u_3 \tag{1.12}$$

[(1.10) is the equation of motion, (1.11) are the components of the electromagnetic field, and (1.12) are the boundary conditions on L_j ($j = 1, 2, \ldots, k$)].

Below, we consider the problem of antiplane deformation (1.10)–(1.12) in the case of a conducting half-space $x_2 \geqslant 0$ with tunnelling crack-cuts L_j along the x_3 axis (Fig. 1). Let the half-space be free from forces and bounded by a vacuum. The static magnetic field in the vacuum is $(0, H_0^*, 0)$ while that in the medium is $(0, H_0, 0)$, where $H_0 = \mu_0 H_0^*/\mu_e$. As the excitation mechanism, we shall take either a shear load $X_{3n} = \mathrm{Re}\,(X_3 e^{-i\omega t})$ which acts on the surfaces of the cavities or a magneto-elastic shear displacement wave which is incident from infinity

$$u_3^\circ = \mathrm{Re}\,(U_3^\circ e^{-i\omega t}),\quad U_3^\circ = \tau\exp\{-i\gamma(x_1\cos\beta + x_2\sin\beta)\} \tag{1.13}$$
$$\gamma_2 = \omega/c_2,\quad \gamma = \gamma_2/\sqrt{1+\lambda^2\sin^2\beta},\quad \tau = \mathrm{const}$$

We shall assume below that the excitation frequency ω is not too large. Thermal effects can then be neglected.

Under these conditions in a body with cracks, there is a stationary (oscillatory) wave process and the components of the fields σ_{3j}, t_{3j} ($j = 1, 2$) and h_3 possess a characteristic root singularity at the vertices of the defects which leads to the need to take account of the influence of electromagnetic effects on the stress intensity factor.

The mechanical wave in a half-space with a defect is made up of the incident wave field (1.13), the field of the reflected wave

$$u_3^{(s)} = \mathrm{Re}\,(U_3^{(s)} e^{-i\omega t}),\quad U_3^{(s)} = \tau\exp\{-i\gamma(x_1\cos\beta - x_2\sin\beta)\} \tag{1.14}$$

and the scattered wave field which, by generalizing [4], we can represent in the form

$$
\begin{gathered}
U_3(x_1, x_2) = \frac{1}{2} \int_L p(\zeta) \left\{ \frac{\partial}{\partial \zeta_1} E(\zeta_1; z_1)\, d\zeta_1 - \frac{\partial}{\partial \bar{\xi}_1} E(\zeta_1; z_1)\, d\xi_1 \right\} + \\
+ \int_L q(\zeta) E(\zeta_1; z_1)\, ds \\
E(\zeta_1; z_1) = H_0^{(1)}(\gamma_2 r_1) + H_0^{(1)}(\gamma_2 r_1^*) \\
\zeta_1 = \xi_1 + \frac{i\xi_2}{\sqrt{1+\chi^2}}, \quad z_1 = x_1 + \frac{ix_2}{\sqrt{1+\chi^2}} \\
\zeta = \xi_1 + i\xi_2 \in L = \bigcup L_j,\ r_1 = |\zeta_1 - z_1|,\ r_1^* = |\xi_1 - z_1|
\end{gathered} \tag{1.15}
$$

Here, $p(\zeta) = \{p_j(\zeta), \zeta \in L_j\}$, $q(\zeta) = \{q_j(\zeta), \zeta \in L_j\}$ are the unknown "densities", $H_0^{(1)}(x)$ is a Hankel function of the first kind of non-zero order and ds is an element of an arc of the contour L. The density $p(\zeta)$ has a simple mechanical meaning: $p(\zeta) = -\frac{1}{2}[U_3(\zeta)]$ where $[U_3(\zeta)]$ is the jump in the displacement amplitude on L.

The function $u_3 = \mathrm{Re}(U_3 e^{-i\omega t})$, where U_3 is defined in (1.15), is the solution of Eq. (1.10) and automatically satisfies the condition $\sigma_{32} = 0$ on the boundary of the half-space as well as the radiation conditions.

2. THE INTEGRAL EQUATION OF THE BOUNDARY VALUE PROBLEM. THE STRESS INTENSITY FACTOR

Taking account of (1.3), we can represent the boundary condition (1.12) in the form

$$
\begin{gathered}
c(\psi) \left\{ \frac{\partial}{\partial z_1} (U_3 + U_3^\circ + U_3^{(s)}) \right\}^{\pm} + \overline{c(\psi)} \left\{ \frac{\partial}{\partial \bar{z}_1} (U_3 + U_3^\circ + U_3^{(s)}) \right\}^{\pm} = \pm X_3^{\pm} \\
X_3^+ = -X_3^- = X_3, \quad c(\psi) = \cos\psi + \frac{i \sin\psi}{\sqrt{1+\chi^2}}
\end{gathered} \tag{2.1}
$$

The upper sign corresponds to the left edge of L_j (when the motion is from its start a_j to the end b_j) and ψ is the angle between the positive normal to the left edge and the x_1 axis.

In accordance with (2.1), we require that the mechanical stress vector should be continuous across the cuts. On carrying out the operations specified in (2.1), we find the relation between the densities

$$
q(\zeta) = \frac{1}{4i} \frac{\chi^2 \sin 2\psi}{\sqrt{1+\chi^2}} \frac{dp}{ds} \tag{2.2}
$$

By virtue of (2.2), it is sufficient to satisfy the boundary condition on one of the edges of L_j.

On substituting the limiting values of the functions occurring here into the boundary condition, we arrive at the following singular integro-differential equation:

$$
\begin{gathered}
\int_L \frac{df}{ds} g(\zeta; \zeta_0)\, ds + \int_L f(\zeta) G(\zeta; \zeta_0)\, ds = N(\zeta_0) \\
g(\zeta; \zeta_0) = \frac{1}{\pi} \mathrm{Im} \left(\frac{c(\psi_0)}{\zeta_1 - \zeta_{10}} \right) + \frac{i\gamma_2 \chi^2 \sin 2\psi}{4\sqrt{1+\chi^2}} \{ H_1(\gamma_2 r_{10}) \mathrm{Re}(c(\psi_0) e^{-i\alpha_{10}}) + \\
+ H_1^{(1)}(\gamma_2 r_{10}^*) \mathrm{Re}(c(\psi_0) e^{-i\alpha_{10}^*}) \} + \frac{\chi^2 \sin 2\psi}{2\pi \sqrt{1+\chi^2}} \mathrm{Re} \left(\frac{c(\psi_0)}{\zeta_1 - \zeta_{10}} \right) \\
G(\zeta; \zeta_0) = \frac{\gamma_2^2}{4i} \{ H_2(\gamma_2 r_{10}) \mathrm{Im}(\overline{a(\psi)}\, \overline{c(\psi_0)} e^{2i\alpha_{10}}) + H_0^{(1)}(\gamma_2 r_{10}) \mathrm{Im}(a(\psi) \overline{c(\psi_0)}) + \\
+ H_0^{(1)}(\gamma_2 r_{10}^*) \mathrm{Im}(a(\psi) c(\psi_0)) + H_2^{(1)}(\gamma_2 r_{10}^*) \mathrm{Im}(\overline{a(\psi)} c(\psi_0) e^{-2i\alpha_{10}^*}) \} \\
N(\zeta) = \frac{2}{\mu} X_3 + 2i\tau\gamma \{ \cos(\psi - \beta) \exp(-i\gamma(\xi_1 \cos\beta + \xi_2 \sin\beta)) +
\end{gathered} \tag{2.3}
$$

$$+\cos(\psi+\beta)\exp(-i\gamma(\xi_1\cos\beta-\xi_2\sin\beta))\}$$
$$2p(\zeta)=-f(\zeta)=-[U_3(\zeta)],\ \zeta_0\in L_j\ (j=1,2,\ldots,k)$$
$$r_{10}^*=|\bar{\xi}_1-\zeta_{10}|,\ r_{10}=|\zeta_1-\zeta_{10}|$$
$$\alpha_{10}^*=\arg(\bar{\xi}_1-\zeta_{10}),\ \alpha_{10}=\arg(\zeta_1-\zeta_{10})$$

The integral equation has to be solved simultaneously with the supplementary conditions

$$\int_{L_j} df=0\quad (j=1,2,\ldots,k) \tag{2.4}$$

Relationships (2.3) and (2.4) completely define the solution in the class h_0 of functions which are unbounded on the ends of L_j [5].

We will now obtain a formula for determining the mechanical stress factor at the vertices of a defect. In order to do this, we parametrize the contour L_j: $\zeta=\zeta(\delta)$, $\zeta_0=\zeta(\delta_0)$, $-1\leqslant\delta,\ \delta\leqslant1$. In accordance with this, we put

$$\frac{df}{ds}=\frac{\Omega(\delta)}{s'(\delta)\sqrt{1-\delta^2}},\quad s'(\delta)=\frac{ds}{d\delta},\quad \Omega(\delta)\in H[-1;1] \tag{2.5}$$

The singular part of the stress σ_n in the continuation beyond the vertex of a defect is determined from (1.15) and (1.3) and has the form

$$\sigma_n=\sigma_{13}\cos\psi+\sigma_{23}\sin\psi=\operatorname{Re}(S_ne^{-i\omega t}) \tag{2.6}$$
$$S_n=\frac{\mu}{2\pi}\int_L\frac{df}{ds}\left\{\operatorname{Im}\left(\frac{c(\psi_c)}{a(\psi)}\frac{d\zeta_1}{\zeta_1-z_1}\right)+\right.$$
$$\left.+\frac{d(\psi)}{2}\operatorname{Re}\left(\frac{c(\psi_c)}{a(\psi_c)}\frac{d\zeta_1}{\zeta_1-z_1}\right)\right\},\quad d(\psi)=\frac{\chi^2\sin2\psi}{\sqrt{1+\chi^2}}$$

where ψ_c is the angle of the normal to the left edge of L_j at the vertex c ($c=a$ or b). An asymptotic analysis of this equation, taking account of formula (2.5), yields (the lower sign refers to the vertex $c=q$)

$$S_n=-\frac{\mu}{2}\frac{\Omega(\mp1)}{\sqrt{2\pi s'(\mp1)}}\left(\sqrt{1+\chi^2}+\frac{\chi^4\sin^22\psi_c}{4\sqrt{1+\chi^2}}\right)(1+\chi^2\sin^2\psi_c)^{-1} \tag{2.7}$$
$$K_{\text{III}}=\lim_{r\to0}\sqrt{2\pi r}\operatorname{Re}(S_ne^{-i\omega t})$$

Hence, a redistribution of the stresses σ_{3j} ($j=1,2$) at the vertex of a defect occurs when there is a preliminary static magnetic field compared with the situation when there is no magnetic field.

The overall stress intensity factor, which takes account of both the mechanical as well as the Maxwellian part of the stress tensor, is defined in terms of the singular part of the expression

$$Q_n=(S_{13}+T_{13})\cos\psi+(S_{23}+T_{23})\sin\psi,\quad t_{ij}=\operatorname{Re}(T_{ij}e^{-i\omega t}) \tag{2.8}$$

By introducing here the first three relationships (1.5) and the formula $t_{3j}=\mu_e(H_3^\circ h_j+H_j^\circ h_3)$ we find

$$Q_n=\mu(\partial_1U_3\cos\psi+(1+\chi^2)\partial_2U_3\sin\psi)$$

By taking account of the asymptotic form Q_n, we obtain the formula for the overall stress intensity factor

$$K_{\text{III}}^{(s)}=\lim_{r\to0}\sqrt{2\pi r}\operatorname{Re}(Q_ne^{-i\omega t})=-\frac{\mu\sqrt{\pi(1+\chi^2)}}{2\sqrt{s'(\mp1)}}|\Omega(\mp1)|\cos(\omega t-\arg\Omega(\mp1)) \tag{2.9}$$

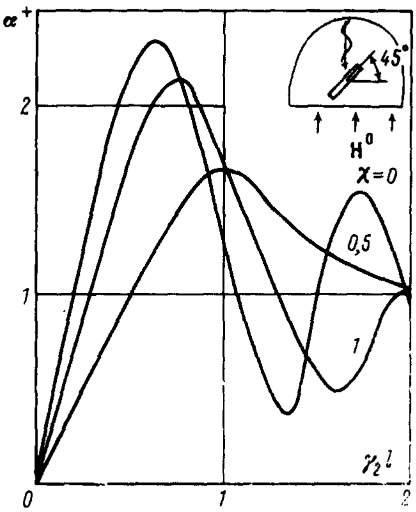

FIG. 2.

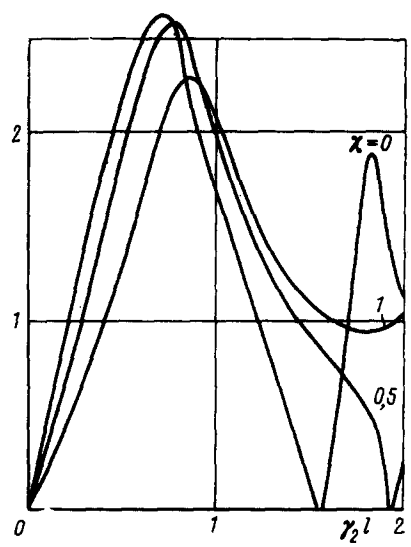

FIG. 3.

3. EXAMPLES

Example 1. Let the half-space be weakened by a rectilinear crack orientated at an angle $\beta_0 = 45°$ to the x_1 axis, the edges of which are free from forces, while a magneto-elastic wave (1.13) is incident along the vertical axis from infinity. The stress intensity factor can be represented in the form

$$K_{III} = P_h \sqrt{l\pi}\, \alpha^{\mp} \cos(\omega t - \arg \alpha^{\mp})$$
$$P_h = -\frac{i\mu\tau\gamma_2 \sin(\beta - \beta_0)}{\sqrt{1 + \chi^2 \sin^2(\beta - \beta_0)}} \quad (3.1)$$

where $\beta_0 = 0°$, $90°$ and $45°$ for horizontal, vertical and inclined cracks respectively and $2l$ is the length of the crack.

The change in the magnitude of α^+ as a function of the normalized wave number for various values of the

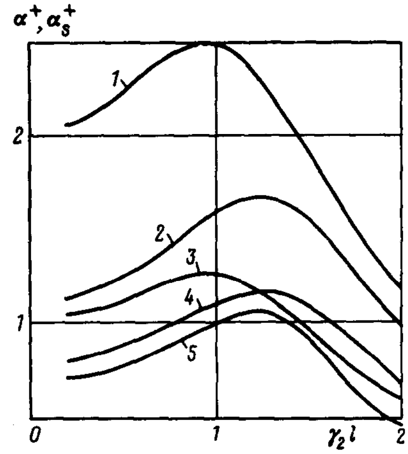

FIG. 4.

parameter χ is shown in Fig. 2. It was assumed that the centre of the crack is at a distance $p_1 = 2l$ from the boundary of the half-space. In the case of a horizontal crack ($p_1 = 2l$), the corresponding results are shown in Fig. 3.

Example 2. Let the space be weakened by a parabolic cut $\xi^1 = p_1\delta$, $\xi_2 = p\delta^2 (-1 \leqslant \delta \leqslant 1)$ and let a magneto-elastic wave (1.13) from infinity be incident along the vertical axis. The total stress intensity factor, $K_{\rm III}$, may be represented in the form of (3.1) where, instead of $\alpha^{\mp}$, there will be $\alpha_s^{\mp}$.

The results of the calculation of the quantities $\alpha = \alpha^{\mp}$ and $\alpha_s = \alpha^{\mp}$ for $p_1 = 1$ are shown in Fig. 4. Curves 1 and 3 were constructed for a straight crack ($p = 0$) and curves 2, 4 and 5 were constructed for a parabolic crack ($p = 1$). Curve 5 corresponds to the value $\theta = 0$ while the remaining curves correspond to $\theta = 1$. Graphs 1 and 2 illustrate the change in α_s, while the remaining graphs show the change in the quantity α.

It follows from the results which have been presented that the influence of electromagnetic effects on the strength of conducting bodies can be very significant.

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